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A Comparative Analysis of LS and MMSE Channel Estimation Techniques for MIMO-OFDM System

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ABSTRACT
Multiple transmit and receive antennas can be used to form multiple-input multiple-output (MIMO) channels to increase the capacity by a factor of the minimum number of transmit and receive antennas. In this paper, orthogonal frequency division multiplexing (OFDM) for MIMO channels (MIMO-OFDM) is considered for wideband transmission to mitigate intersymbol interference and enhance system capacity. In this paper performance analysis of channel estimation through different algorithms for estimating channel using BPSK modulation scheme are investigated for different channel delay spread. The estimation of channel at pilot frequencies is based on Least Square, Minimum mean square channel estimation algorithm. We have compared the performances of these two channel estimation algorithm by measuring bit error rate Vs SNR. Minimum Mean Square estimation has been shown to perform much better than Least Square channel estimation algorithm.
Keywords – Delay Profile, Channel Estimation, Least Square, MIMO-OFDM, Minimum Mean Square

I. Introduction

It is a well-known fact that the amount of information transported over communication systems grows rapidly. Not only the file sizes increase, but also large bandwidth-required applications such as video on demand and video conferencing require increasing data rates to transfer the information in a reasonable amount of time or to establish real-time connections. To support this kind of services, broadband communication systems are required. Large-scale penetration of wireless systems into our daily lives will require significant reductions in cost and increases in bit rate and/or system capacity. Recent information theoretical studies have revealed that the multipath wireless channel is capable of huge capacities, provided that multipath scattering is sufficiently rich and is properly exploited through the use of the spatial dimension. Appropriate solutions for exploiting the multipath properly, could be based on new techniques that recently appeared in literature, which are based on Multiple Input Multiple Output (MIMO) technology. Basically, these techniques transmit different data streams on different transmit antennas simultaneously. By designing an appropriate processing architecture to handle these parallel streams of data, the data rate and/or the Signal-to-Noise Ratio (SNR) performance can be increased. Multiple Input Multiple Output (MIMO) systems are often combined with a spectrally efficient transmission technique called Orthogonal Frequency Division Multiplexing (OFDM) to avoid Inter Symbol Interference (ISI).[1]

Channel estimation is a crucial and challenging issue in coherent demodulation. Its accuracy has significant impact on the overall performance of the MIMO-OFDM system. The digital source is usually protected by channel coding and interleaved against fading phenomenon, after which the binary signal is modulated and transmitted over multipath fading channel. Additive noise is added and the sum signal is received. Due to the multipath channel there is some inter-symbol interference (ISI) in the received signal. Therefore a signal detector needs to know channel impulse response (CIR) characteristics to ensure successful removal of ISI.

The channel estimation in MIMO-OFDM system is more complicated in comparison with SISO system due to simultaneous transmission of signal from different antennas that cause co-channel interference. This issue highlights that developing channel algorithm with high accuracy is an essential requirement to achieve full potential performance of the MIMO-OFDM system. A considerable number of channel estimation methods have already been studied by different researchers for MIMO systems. A wideband channel is normally frequency selective and time variant. For an MIMO OFDM mobile communication system, the channel transfer function at different subcarriers appears unequal in both frequency and time domains. Therefore, a dynamic estimation of the channel is necessary. Pilot-based approaches are widely used to estimate the channel properties and correct the received signal. There are two types of pilot arrangements: Block type and Comb type.[2] In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all sub-carriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error since the pilots are sent at all carriers. fast fading channel, where the channel changes between adjacent OFDM symbols, the pilots are transmitted at all times but with an even spacing on the subcarriers, representing a comb type pilot channel estimation [3].

Several Channel Estimation techniques have been proposed to mitigate interchannel interference (ICI) in OFDM systems. The least square (LS) CE has been proposed to minimize the squared differences between the received and estimated signal. The LS algorithm, which is independent of the channel model, is commonly used in equalization and filtering applications. But the statistics of channels in real world change over time and inversion of the large dimensional square matrix turns out to be ill-conditioned. To further improve the accuracy of the estimator, Wiener filtering based iterative CE has been investigated; However, this scheme also requires high complexity and knowledge of channel correlations. The most important research topic in the wireless communications is the adaptive CE where the channel is rapidly time-varying. The time-varying multipath channel can be represented by a tap delayed line with time varying coefficients and fixed tap spacing. An adaptive algorithm is a process that changes its parameters as it gain more information of its possibly changing environment. Among numerous iterative techniques that exist in the open literature, the popular category of approaches which are obtain from the minimization of the mean square error (MSE) between the output of the filter and desired signal to perform CE. MIMO OFDM systems normalized least mean (NLMS) square and recursive least squares (RLS) adaptive channel estimator are described for MIMO OFDM systems)[4].

II. MIMO-OFDM System

We consider MIMO–OFDM systems with two transmit antennas and two receive antenna as shown in Fig.1

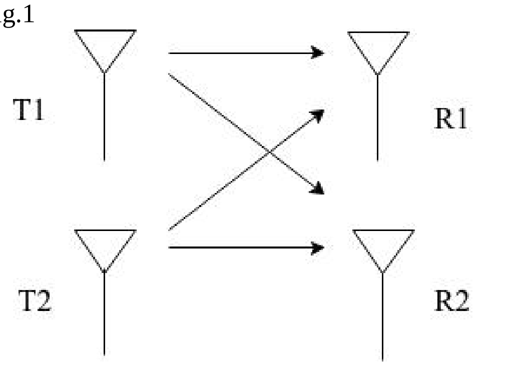

Fig. 1 MIMO-OFDM System with two transmitting antenna and two receiving antenna

The total number of subcarriers is N. Basically; the MIMO-OFDM transmitter has Nt parallel transmission paths which are very similar to the single antenna OFDM system. In OFDM system the binary data is first grouped and mapped according to the modulation in “signal mapper”. After modulation the symbol rate reduced to R = (R /log2M), where M is constellation size. Then this serial data is fed to serial to parallel convertor. This reduces data rate by N times, where N is number of parallel streams. Each of parallel streams constitutes tiny bandwidth in the spectrum. So these streams almost undergo flat fading in the channel. After inserting pilots either to all subcarriers with a specific period of blocks or within a uniform period of frequency bins in all blocks, IDFT block is used to transform the data sequence of length into time domain signal $x(n) = IDFT\{X(k)\}$, n=0, 1, 2, ………N-1

$$= \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}} \tag{1}$$

At the receiver’s end, after passing to discrete domain through A/D and low pass filter, guard time is removed and the signal $Y^g(n+n_g)$ for $-N_g \le n \le N-1$ will become as y(n) for n=0,1,2,……N-1

Then y (n) is sent to DFT block for the following operation:

$$Y(k) = DFT\{y(n)\} \tag{2}$$

k=0, 1……N-1

As a matter of convenience we can write the entire operation as

$$Y(k) = X(k)H(k) + W(k) \tag{3}$$

Where,

$X(k) = DFT\{x(n)\}$ and $W(k) = DFT\{w(n)\}$

Then the binary information data is obtained back in “signal Demapper” block, where *N* is DFT length. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time is a copy of the last part which is prepended to OFDM symbol. This makes the transmitted symbol periodic, which plays a key role in identifying frames correctly, so as to avoid ISI and intercarrier interference (ICI). The resultant OFDM symbol is given as follows:

$$\underline{X}(n) = x(n_g + n) \quad (4)$$

$n_g = N_g, N_g+1, \ldots\ldots.-1$

Where Ng is the length of the guard interval. The transmitted signal will pass through the frequency selective time varying fading channel with additive noise. Then the received signal is

$$Y(n) = \underline{X}(n)H(n) + W(n) \quad (5)$$

As a MIMO signalling technique, *Nt* different signals are transmitted simultaneously over $N_t \times N_r$ transmission paths and each of those *Nr* received signals is a combination of all the *Nt* transmitted signals and the distorting noise. It brings in the diversity gain to enhance system capacity as we desire. The data stream from each antenna undergoes OFDM Modulation with the encoding matrix represented as

$$X = \begin{bmatrix} X1 & -X2^* \\ X2 & X1^* \end{bmatrix} \quad (6)$$

$$X1 = X[0] - X^*[1]X[2] - \ldots\ldots\ldots X^*[N-1]$$
$$X2 = X[1] - X^*[0]X[3] - \ldots\ldots\ldots X^*[N-2]$$

The vectors $X1$ and $X2$ are modulated using the inverse fast Fourier transform (IFFT) and after adding a cyclic prefix as a guard time interval, two modulated blocks $Xg1$ and $Xg2$ are generated and are then transmitted by the first and second Transmit antennas respectively. Assuming that the guard time interval is more than the expected largest delay spread of a multipath channel. The received signal will be the convolution of the channel and the transmitted signal. Assuming that the channel is static during an OFDM block, at the receiver side after removing the cyclic prefix, the FFT output as the demodulated received signal can be expressed as:

$$\Rightarrow \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_r \end{bmatrix} = \begin{bmatrix} H_{11}H_{12}\cdots H_{1t} \\ H_{21}H_{22}\cdots H_{2t} \\ \vdots\ddots\ddots\ddots\ddots\vdots \\ H_{r1}H_{r2}\cdots H_{rt} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_t \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_r \end{bmatrix} \quad (7)$$

In the equation 7, [$W1$, $W2$……WNT] denotes AWGN and $H_{m,n}$ is the (single-input single-output) channel gain between the mth receive and nth transmit antenna pair. The nth column of **H** is often referred to as the spatial signature of the nth transmit antenna across the receive antenna array.
Knowing the channel information at the receiver, Maximum Likelihood (ML) detection can be used for decoding of received signals for two antenna transmission system, which can be written as:

$$\overline{S}[2k] = \sum_{i=1}^{Nk} H^*_{i,1}[2k]Y_i[2k] + H_{i,2}[2k]Y^*_i[2k+1] \quad (8)$$

$$\overline{S}[2k+1] = \sum_{i=1}^{Nk} H^*_{i,2}[2k+1]Y_i[2k] - H_{i,1}[2k]Y^*_i[2k+1] \quad (9)$$

Where k=0, 1, 2…………. (N/2)-1
Assuming that the channel gains between two adjacent sub channels are approximately equal i.e.

$$H_{i,1}[2k] = H_{i,1}[2k+1]$$
$$H_{i,2}[2k] = H_{i,2}[2k+1]$$

At the end, the elements of block s[k]are demodulated to take out the information data.

III. Training Sequence Based Channel Estimation

Based on the asumptions such as perfect synchronization and block fading, a MIMO-OFDM system is design. In training based channel estimation algorithms, training symbols or pilot tones that are known to the receiver, are multiplexed along with the data stream for channel estimation [5]. The idea behind these methods is to develop knowledge of transmitted pilot symbols at the receiver to estimate the channel. For a block fading channel, where the channel is constant over a few OFDM symbols, the pilots are transmitted on all subcarriers in periodic intervals of OFDM blocks. The channel estimates from the pilot subcarriers are interpolated to estimate the channel at the data subcarrier. This type of pilot arrangement, given in Fig.2 is called the block type arrangement

In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all subcarriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error since the pilots are sent at all carriers. For a fast fading channel, where the channel changes between adjacent OFDM symbols, the pilots are transmitted at all times but with an even spacing on the subcarriers, representing a comb type pilot placement, This type of pilot arrangement, given in Fig.2 is called the comb type arrangement The estimation can be performed by using either LS or MMSE.

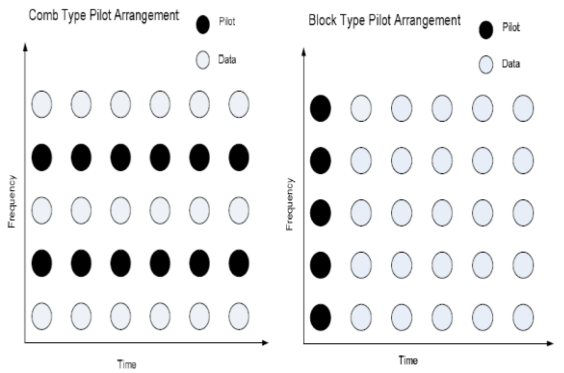

Fig.2 Block Type & Comb Type Pilot Arrangement

3.1 Least Squares Channel Estimation

Defining symbols as in [26], $\vec{X} = [X_k]^T$ and $\vec{Y} = [Y_k]^T$, $k = 0,1,\ldots.N-1$ and X_k and Y_k are the transmitted symbols and N is the block length of the transmitted symbols. Let $\vec{g} = [g_k]^T$ and $\vec{n} = [n_k]^T$ denote the channel impulse response and AWGN respectively. Let the input matrix $\underline{X} = diag(\vec{X})$ and the DFT matrix be

$$\underline{F} = \begin{bmatrix} W^{00} & \cdots & W^{0(N-1)} \\ \vdots & \ddots & \ddots \\ W^{(N-1)0} & \cdots & W^{(N-1)(N-1)} \end{bmatrix} \qquad (10)$$

where $W^{l,k} = \frac{1}{\sqrt{N}} e^{\frac{-j2\pi k}{N}}$, From here let $\vec{H} = DFT_N(\vec{g})$ and $\vec{N} = DFT_N(\vec{n})$. The received signal $\vec{Y}$ is given by $\vec{Y} = \underline{X}\vec{H} + \vec{N}$. The LS estimator attempts to minimize the quantity $(\vec{Y} - \underline{X}\vec{H})^H (\vec{Y} - \underline{X}\vec{H})$ where $(\bullet)^H$ denotes complex conjugate transpose. The least square channel estimate is given by

$$H_{LS} = \underline{X}^{-1}\vec{Y} = \left[\frac{X_k}{Y_k}\right]^T \qquad (11)$$

The LS estimator has low complexity and thus can be easily implemented but it suffers from high mean square error since it aims to minimize the least square error.

3.2 Minimum Mean Squares Error (MMSE) Channel Estimation

The MMSE estimator uses second order statistics which involve using the channel auto covariance in order to minimize the square error. Here the channel second order statistics are assumed to be known at the receiver. This means the auto covariance of the channel impulse response (R_{gg}) and transfer function $(\underline{R}_{HH})$ are known. Defining

$$\underline{R}_{HH} = E\left\{\vec{H}\vec{H}^H\right\} = E\left\{\left(\underline{F}\vec{g}\right)\left(\underline{F}\vec{g}\right)^H\right\} = \underline{F}\,\underline{R}_{gg}\,\underline{F}^H$$

$$\underline{R}_{gY} = E\left\{\vec{g}\vec{Y}^H\right\} = E\left\{\vec{g}\left(\underline{X}\,\underline{F}\vec{g} + \vec{N}\right)^H\right\} = \underline{R}_{gg}\,\underline{F}^H\,\underline{X}^H$$

$$\underline{R}_{YY} = E\left\{\vec{Y}\vec{Y}^H\right\} = \underline{X}\,\underline{F}\,\underline{R}_{gg}\,\underline{F}^H\,\underline{X}^H + \delta_N{}^2 I_N$$

MMSE estimator of $\vec{H}$ is given by

$$\begin{aligned} \underline{\vec{H}}_{MMSE} &= \underline{F}\,\underline{R}_{gY}\,\underline{R}_{YY}{}^{-1}\vec{Y} = \underline{F}\left[\left(\underline{F}^H\,\underline{X}^H\right)^{-1}\underline{R}_{gg}{}^{-1}\delta_N{}^2 + \underline{X}\,\underline{F}\right]^{-1}\vec{Y} \\ &= \underline{F}\,\underline{R}_{gg}\left[\left(\underline{F}^H\,\underline{X}^H\,\underline{X}\,\underline{F}\right)^{-1}\delta_N{}^2 + R_{gg}\right]\underline{F}^{-1}H_{LS} \\ &= \underline{R}_{HH}\left[\delta_N{}^2\left(\underline{X}\,\underline{X}^H\right)^{-1} + \underline{R}_{HH}\right]^{-1}H_{LS} \end{aligned} \qquad (12)$$

As can be seen the MMSE estimator is much more complex than the LS estimator. It involves more multiplications and matrix inversion. The modified MMSE reduces the complexity of the MMSE estimator by using methods such as singular value decomposition.

IV. Simulation And Results

4.1 Simulation Parameters

In this thesis, in order to understand the effects of applying different channel estimation techniques on a MIMO-OFDM system, the system have been simulated using Matlab.

In our simulation, we use Rayleigh multipath propagation channel and additive white Gaussian noise with zero mean and variance determined by the SNR is considered. The channels corresponding to different transmit or receive antennas have the same statistics. Two transmit antennas and two receive antennas are used to form a two-input two-output OFDM system.

To construct an OFDM signal, we assume the entire channel bandwidth 1.25 MHz is divided into 256 sub-channels. The two sub-channels on each end are used as guard tones and the rest are used to transmit data. To make the tones orthogonal to each other, the symbol duration is about 204.8 µs. An additional 20.2 µs guard interval is used to provide protection from ISI due to channel multipath delay spread. This results in a total block duration $T_f = 204.8 + 20.2 = 225\,\mu s$ and a sub-channel symbol rate $r_b = 4.44\,kBd$. Each data block, containing 500 information bits, is coded into two different blocks, each of which has exactly 252 symbols, to form an OFDM block. Therefore, the OFDM system with two transmit antennas can transmit a data block in parallel. Each time slot consists of ten OFDM blocks, with the first block used for training and the following nine blocks used for data transmission. Consequently, the described system can transmit at 2 Mb/s over a 1.25 MHz channel, i.e., the transmission efficiency is 1.6 b/s/Hz.

4.2 Simulation Results

In Fig.3 & Fig.4 the BER performance comparison of LS and MMSE estimation techniques is performed for a 2×2 MIMO-OFDM system for different channel tapping lengths L=4, 8, 16 over AWGN channel. The BER performance for system improves by increasing the channel tapping length, but by increasing tap length the complexity of system increases which in turn increases the complexity of these two estimation techniques. In order to limit computational complexity, constraint length has to be restricted below a certain limit. By increasing the tap length beyond a certain limit no significant improvement in BER performance can be achieved. The BER performance of the MIMO-OFDM system is better for MMSE as compared to LS estimation technique for different tap lengths.

Fig.4 shows the BER performance comparison for Minimum Mean Square Error (MMSE) estimation technique for channel tapping lengths L=4, 8, 16 for 2×2 MIMO-OFDM over an AWGN channel. It is clearly seen from the Fig.3 that by increasing the tap length BER performance of the system improves. On applying MMSE estimation technique one can achieve the BER 10^{-4} at 16.2dB, 17.4dB, 19.5dB for tap lengths 16, 8, 4 respectively.

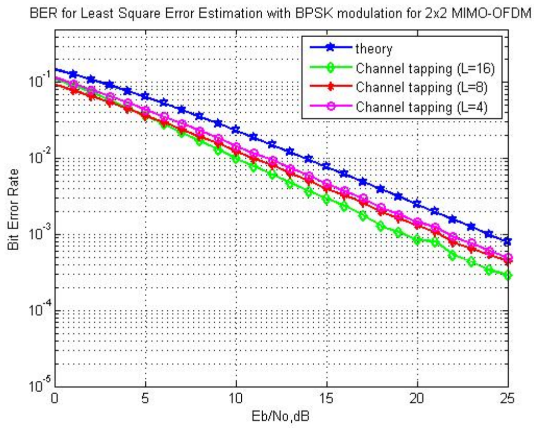

Fig. 3 BER performance comparisons for different channel tapping lengths by using Least Square (LS) estimation technique for 2×2 MIMO-OFDM

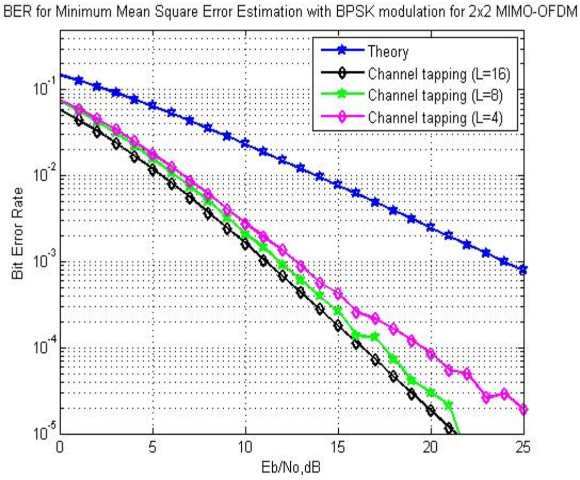

Fig.4 BER performance comparisons for different channel tapping lengths by using Minimum Mean Square (MMSE) estimation technique for 2×2 MIMO-OFDM

V. Conclusion

Channel estimation allows the receiver to compensate for variations in the communication channel in order to reduce the probability of error in the detected signal. The estimation is obtained by sending reference signals called pilots which are known at the receiver but the disadvantage of using them is that the data rate is reduced since less information is transmitted because pilots are known at the receiver and hence they are not information. From this it seems that there is loss of data throughput but by using MIMO-OFDM this loss is very small as compared to the increased capacity of the system. From Fig.3 & Fig.4 it is clear that the BER performance for MMSE estimate has been shown to be better than the LS estimate for channel estimation in MIMO-OFDM systems based on block-type pilot arrangement. The BER for MMSE estimate has about 5-15 dB gain in SNR over the LS estimate for the same BER values. The major drawback of the MMSE estimate is its high complexity. Also, on increasing the tap length the BER performance increases for both estimation techniques but as the tap length value becomes high there is minimal increment in BER performance.

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